

Semester 1 (Unit 3) Examination, 2019

Question/Answer Booklet

MATHEMATICS METHODS

Section One: Calculator-free

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	13	13	100	100	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the School/College assessment policy. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free**35% (50 Marks)**

This section has **(seven) 7** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Suggested working time: 50 minutes

Question 1**(11 marks)**

(a) Differentiate xe^{3x} with respect to x .

(2 marks)

(b) Determine $\frac{d}{dx}\left(\frac{\cos x}{x^3}\right)$. Simplify your answer.

(3 marks)

(c) Let $g(u) = \sqrt{u}$ and $u = 2 - 3x^2$.

Demonstrating clear use of the chain rule, determine $\frac{dg}{dx}$.

(3 marks)

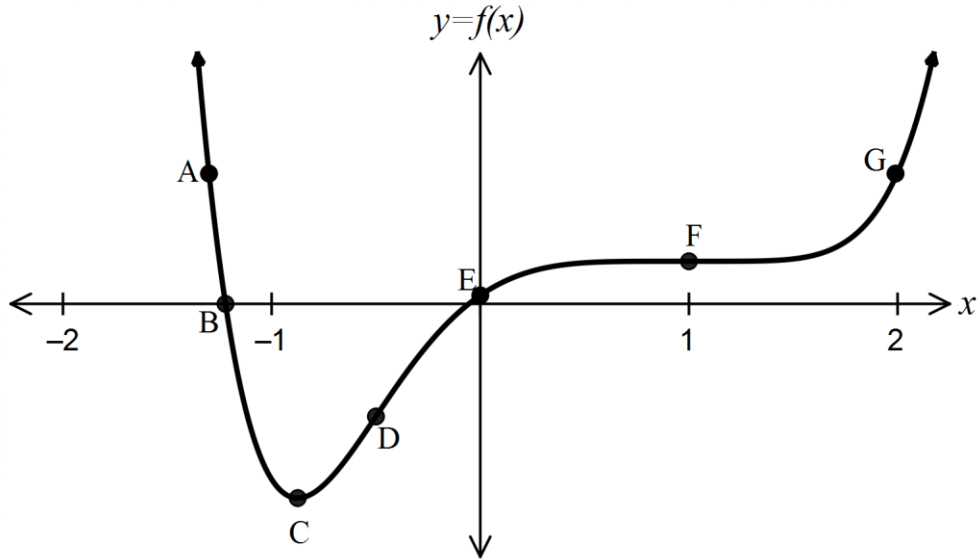
(d) A particle moves in a straight line such that its displacement, $x(t)$ from the origin is given by $x(t) = 3\sin 2t$ where x is measured in metres and t is measured in seconds. Use calculus to determine when the particle first stops.

(3 marks)

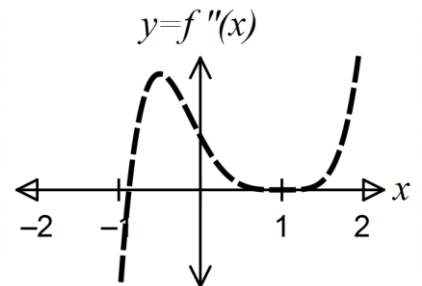
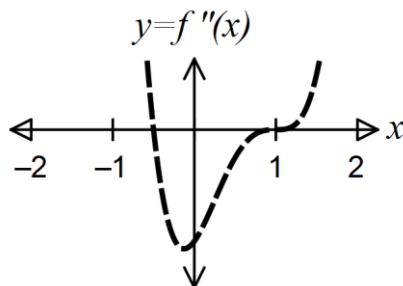
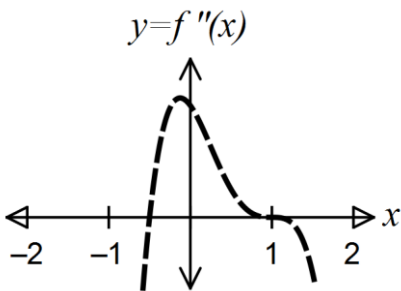
Question 2

(5 marks)

Consider the function $y = f(x)$ shown below. The points, A, B, C, D, E, F and G each lie on the graph.



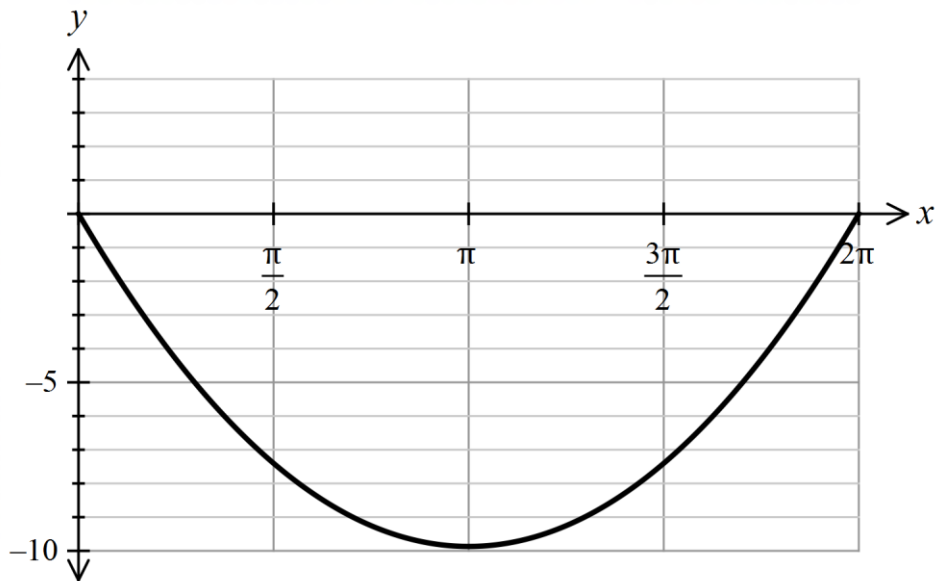
- (a) Which point/s labelled on the graph above satisfy the following,
- (i) a point of inflection occurs? (1 mark)
 - (ii) $f'(x) = 0$ and $f''(x) \neq 0$? (1 mark)
 - (iii) f is increasing and $f''(x) < 0$? (1 mark)
 - (iv) $f(x) > 0$ and the function is concave up? (1 mark)
- (b) Circle the graph below that represents $f''(x)$. (1 mark)



Question 3

(4 marks)

The graph of $g(x) = x(x - 2\pi)$, over the domain $0 \leq x \leq 2\pi$ is drawn below.



Calculate the exact area between the two curves, $f(x) = \sin x$ and $g(x) = x(x - 2\pi)$, for $0 \leq x \leq 2\pi$.

Question 4**(9 marks)**

(a) Determine $\int \left(2e^{2x} - \frac{3}{\sqrt{x}} \right) dx$. (2 marks)

(b) Evaluate $\int_0^1 (3 - 2x)^2 dx$. (2 marks)

(c) Determine $F'(x)$ if $F(x) = \int_x^1 \frac{dt}{1 + \sqrt{1-t}}$. (2 marks)

(d) Solve for m given $\int_{-m}^m (m^3 - x^3) dx = 1250$. (3 marks)

Question 5**(10 marks)**

Melinda and Matt are two students of Mathematics Methods. They are playing a board game using two unbiased dice. The person who throws the first double six gets to start the game.

Melinda and Matt decide to apply their knowledge of probability distributions to their game. Before Matt throws the two dice for the first time he defines a random variable S . The random variable S is assigned the value 1 if two sixes are thrown and 0 otherwise.

- (a) Describe the distribution of S by stating its name, mean and variance. (3 marks)

Melinda and Matt only have a short time to play the game so decide they will stop after each player has had 15 turns of rolling the dice.

Define the random variable W as the total number of double sixes that Melinda throws in her 15 rolls of the dice.

- (b) Describe the distribution of W including its parameters. (3 marks)

- (c) State an expression to determine the probability that Melinda throws just one pair of double sixes in her 15 throws. (1 mark)

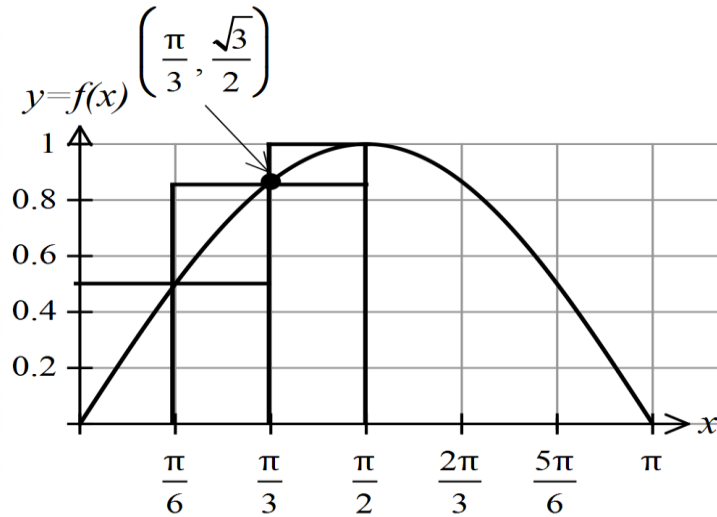
- (d) State an expression to determine the probability that the total number of double sixes thrown by Melinda and Matt in their 15 rolls each of the dice is exactly 2 two given that Melinda threw at least one double six. (3 marks)

Question 6

(4 marks)

The graph below shows the function $y = f(x) = \sin x$ for $0 \leq x \leq \pi$.

Rectangles of width $\frac{\pi}{6}$ are also drawn.



(a) Use these rectangles to determine

(i) an underestimate for the area under $f(x) = \sin x$ from $x=0$ to $x = \frac{\pi}{2}$. (1 mark)

(ii) an overestimate for the area under $f(x) = \sin x$ from $x=0$ to $x = \frac{\pi}{2}$. (1 mark)

- (b) Show that by using three trapeziums an estimate for the area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$ is given by $\frac{\pi}{6} \left(\frac{2 + \sqrt{3}}{2} \right)$.

(2 marks)

Question 7**(7 marks)**

(a) Determine $\frac{dy}{dx}$ where $y = \sin^2 x$.

(1 mark)

(b) Hence show clearly that $\int \sin x \cos x dx = \frac{1}{2} \sin^2 x + c$.

(3 marks)

(c) Hence or otherwise, evaluate $\int_0^{\frac{\pi}{6}} (\sin x \cos x + 2) dx$

(3 marks)

Additional working space

Question number: _____

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